Restructurable Control Using Proportional-Integral Implicit Model Following

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Studies of a proportional-integral implicit model-following control law are presented. The research focuses on the ability of the control law to recover the performance of a system with failed actuators or structural damage to its prefailure level. Properties of the implicit model-following strategy are examined, and conditions for control reconfiguration are stated. The control law is applied to the lateral-directional model of a fighter aircraft, and control restructuring is shown for changes in control and system matrices. It is concluded that the implicit model-following scheme is a good candidate for control reconfiguration.

Introduction

FAULT-TOLERANT control systems typically have one or more of three attributes: robustness, reconfigurability, and restructurability. Robustness implies insensitivity to variations in the parameters or structure of the system. If physical parameters change or the nature of system dynamics is altered, the robust system continues to operate as originally intended, providing response to inputs and disturbances that is close to nominal performance. Reconfigurability implies that some of the control system's parameters can be purposely modified to account for uncontrollable changes in the system, such as failed sensors, actuators, or damaged structural elements. Restructurability subsumes reconfigurability, implying that not only parameters but the structure of the system itself can be changed to accommodate uncontrollable changes. Whereas the robust system obtains fault tolerance with fixed parameters and structure, reconfigurable and restructurable systems are adaptive, providing some degree of fault detection and identification as well as the ability to alter the system. The latter should achieve satisfactory performance over a wider range of conditions than is obtained by robustness alone; otherwise the added complexity is not warranted. This paper presents a linear-quadratic (LQ) control strategy that is robust and that allows control restructuring to be carried out in a natural fashion.

Linear-quadratic control theory inherently provides a number of useful features, such as direct treatment of multi-input/multi-output problems, easy transfer from continuous- to discrete-time descriptions, and guaranteed stability when certain criteria are satisfied.[1] However, the choice of quadratic state- and control-weighting matrices (Q and R) is not always obvious. This problem is of particular concern when a control system is to be reconfigured or restructured in real time following failures. Suppose that the chosen Q and R lead to desirable closed-loop characteristics for the nominal system (described by system and control matrices, F_1 and G_1). If failures cause the open-loop system to change (to F_2 and G_2), a controller designed with the same Q and R will have different closed-loop characteristics. Although the original cost function is, by definition, still optimized, the resulting response may no longer be adequate in a subjective sense. If similar system behavior is desired before and after the failure, it would be necessary in most cases to find new values of Q and R, which may be a nontrivial selection process.

Model following is an attractive candidate for the redesign process because the goal is to emulate the performance characteristics of a desirable model, with or without failures. This can be accomplished explicitly (by requiring the system outputs to follow those of the model in a least-squares sense) or implicitly (by minimizing a quadratic function of the actual and modeled state rates).[2,3] Explicit model following normally is less sensitive to parameter variations, but the model states must be generated, and feedback gains may be high for satisfactory performance. Implicit model following can be implemented more simply with lower gains, and the weighting matrices are directly affected by the difference in plant and model dynamics. Consequently, the effective Q and R of the LQ problem can be adjusted by known changes in plant dynamics so as to minimize the variation in closed-loop performance.

Model-following criteria were discussed by Erzberger,[2] who showed that explicit and implicit model following are equivalent provided certain conditions are satisfied. The synthesis of model-following systems was addressed by Tyler,[3] who analyzed feedback and feed-forward gains in terms of plant and model characteristics. Chan[4] examined the relative merits of explicit and implicit model-following controls and concluded that the former yielded smaller steady-state error when imperfect model following occurred. Asseo[5] formulated a type-one perfect model-following control law and found that it reduced sensitivity to parameter variations. Kreindler and Rothschild[6] also compared both model-following strategies and found that the implicit model-following method tracked the model better, especially during the initial transient. Broussard and Berry[7] considered a different formulation of implicit model following, in which feedback gains are determined from algebraic relationships. They showed that the approach is equivalent to pole-placement techniques. Kaufman[8] employed model-following concepts in the design of adaptive control systems. Digital implicit model following was studied in Ref. 9, and flight test results for a simple algebraic design method are presented in Ref. 10.

The subject of restructurable control is an area of ongoing research. Huber et al.[11] designed a self-repairing control system that utilized a control mixer concept to distribute control authority after a surface failure to remaining effectors. Os-

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troff and Hueschen[12] applied a command-generator-tracker/proportional-integral filter control law to a commercial airplane with control surface failures and under turbulence. Looze et al.[13] reported on an automatic redesign technique that restructures control such that a frequency-domain system performance metric is maximized.

In this paper, a proportional-integral implicit model-following (PIIMF) control law is outlined, and associated features are discussed. Some properties of the implicit model-following scheme and conditions for control reconfiguration, given changes in control and system matrices, are examined. PIIMF control is applied to an example problem based on the lateral-directional motions of a typical fighter aircraft. Automatic restructuring is illustrated for the changes that might be brought about by control actuator failures or aircraft structural and aerodynamic damage. The results are summarized in the conclusion.

Control Law Formulation

The system to be controlled, the desired dynamic model, and the cost function to be minimized are, respectively,

$$\dot{x} = Fx + Gu \tag{1}$$

$$\dot{x}_m = F_m x_m \tag{2}$$

$$J = \frac{1}{2}\int_0^\infty \left[(\dot{x}-\dot{x}_m)^T Q_i (\dot{x}-\dot{x}_m)\right] \mathrm{d}t \tag{3}$$

or

$$J = \frac{1}{2}\int_0^\infty \left[x^T Q x + 2x^T M u + u^T R u\right] \mathrm{d}t \tag{4}$$

where x and u are actual state and control vectors of dimensions $n \times l$ and $r \times l$, respectively, x_m is an $n \times l$ model state vector, F and G are system and control matrices for the system to be controlled, F_m is a model matrix, and Q_i is a weighting matrix for the state-rate errors. Substituting Eqs. (1) and (2) into Eq. (3), and assuming that $x = x_m$, leads to Eq. (4), where $Q=(F-F_m)^T Q_i (F-F_m)$, $M=(F-F_m)^T Q_i G$, and $R = G^T Q_i G$. From Eq. (4), we see that the PIIMF controller generates a cross-weighting matrix M. Considering optimality and pole-assignment requirements,[14,15] the presence of M is essential for achieving complete control over the system.

Equation (4) is simply a quadratic cost function of the original system [(Eq. (1)]. In addition to the inherent sensitivity to plant parameter variations,[5] we can improve the low-frequency disturbance-rejection characteristics of the control law and achieve zero steady-state error command response by adding proportional-integral (PI) compensation. Let y_d be the desired equilibrium outputs; then the perturbations around the corresponding set point (x_0, u_0) are

$$\Delta x(t) = x(t) - x_0 \tag{5}$$

$$\Delta u(t) = u(t) - u_0 \tag{6}$$

$$\Delta y(t) = y(t) - y_d \tag{7a}$$

$$= Hx(t) - y_d \tag{7b}$$

where H is the output matrix. The output error integral ξ is defined as

$$\xi = \xi(0) + \int_0^t \Delta y(\tau)\, \mathrm{d}\tau \tag{8}$$

The state variable is augmented to include ξ,

$$\Delta \dot{x}_a(t) = \left[\Delta \dot{x}^T(t) \quad \xi^T(t)\right]^T = \begin{bmatrix} F & 0 \\ H & 0 \end{bmatrix} \Delta x_a(t) + \begin{bmatrix} G \\ 0 \end{bmatrix} \Delta u(t) \tag{9}$$

and the cost function becomes

$$J = \frac{1}{2}\int_0^\infty \left[\Delta x_a^T Z \Delta x_a + 2\Delta x_a^T S \Delta u + \Delta u^T R \Delta u\right] \mathrm{d}t \tag{10}$$

where

$$Z = \begin{bmatrix} Q & 0 \\ 0 & Q_\xi \end{bmatrix}, \qquad S = \begin{bmatrix} M \\ 0 \end{bmatrix}$$

and Q_ξ is the weighting on ξ. The LQ control obtained by optimizing Eq. (10) is

$$\Delta u(t) = -C\Delta x_a(t) = -[C_1 \quad C_2]\begin{bmatrix} \Delta x(t) \\ \xi(t) \end{bmatrix} \tag{11}$$

Expanding and rearranging Eq. (11), we get

$$u(t) = u_0 - C_1[x(t)-x_0] + C_2\left\{\xi(0) + \int_0^t [y(\tau)-y_d]\, \mathrm{d}\tau\right\} \tag{12}$$

We can see that the second term on the right side provides proportional feedback of the error, whereas the third term provides the integral effect being sought. The dimension of the commanded input y_d is chosen to equal the dimension of u, reflecting the fact that the number of controllable states is equal to the number of the controls available.[1]

Properties of the Control Law

We first examine the effect of adding the integral loop. The steady-state Riccati equation resulting from optimizing Eq. (10) is

$$\begin{aligned}
&\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}\left[\begin{pmatrix} F & 0 \\ H & 0 \end{pmatrix} - \begin{bmatrix} G \\ 0 \end{bmatrix} R^{-1}\begin{bmatrix} M^T & 0 \end{bmatrix}\right] \\
&+ \left[\begin{pmatrix} F & 0 \\ H & 0 \end{pmatrix} - \begin{bmatrix} G \\ 0 \end{bmatrix} R^{-1}\begin{bmatrix} M^T & 0 \end{bmatrix}\right]^T \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \\
&- \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}\begin{bmatrix} G \\ 0 \end{bmatrix} R^{-1}\begin{bmatrix} G^T & 0 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \\
&+ \begin{pmatrix} Q & 0 \\ 0 & Q_\xi \end{pmatrix} - \begin{bmatrix} M \\ 0 \end{bmatrix} R^{-1}\begin{bmatrix} M^T & 0 \end{bmatrix} = 0
\end{aligned} \tag{13}$$

Expanding along each submatrix, we get

$$\begin{aligned}
&P_{11}(F - GR^{-1}M^T) + (F - GR^{-1}M^T)^T P_{11} - P_{11} G R^{-1} G^T P_{11} \\
&+ Q - MR^{-1}M^T + P_{12}H + H^T P_{12}^T = 0
\end{aligned} \tag{14}$$

$$P_{12}^T(F - GR^{-1}M^T) + P_{22}H - P_{12}^T G R^{-1} G^T P_{11} = 0 \tag{15}$$

$$-P_{12}^T G R^{-1} G^T P_{12} + Q_\xi = 0 \tag{16}$$

The Riccati equation obtained by optimizing the implicit model-following strategy alone [Eq. (4)] is:

$$\begin{aligned}
&P(F - GR^{-1}M^T) + (F - GR^{-1}M^T)^T P \\
&- PGR^{-1}G^T P + Q - MR^{-1}M^T = 0
\end{aligned} \tag{17}$$

Therefore, the effect of the integration loop is to add the terms $P_{12}H + H^T P_{12}^T$. Since from Eq. (16) we see that P_{12} is determined by G and Q_ξ, and it is independent of P_{11} and the model, these extra terms constitute a shift or "bias" in state-weighting Q.

We now proceed to the analysis of the implicit model-following strategy, making comments on the integrator effect whenever necessary. If G were invertible (which would require the number of controls to be equal to the number of states) and Q_i were diagonal, then from the definitions of Q, M, and R,

$$(F-GR^{-1}M^T)=F-GR^{-1}\Big[(F-F_m)^T Q_i G\Big]^T \tag{18a}$$

$$=F-GR^{-1}G^TQ_i^T(F-F_m) \tag{18b}$$

$$=F-G(G^TQ_iG)^{-1}G^TQ_i^T(F-F_m) \tag{18c}$$

$$=F-GG^{-1}Q_i^{-1}G^{-T}G^TQ_i(F-F_m) \tag{18d}$$

$$=F-F+F_m \tag{18e}$$

$$=F_m \tag{18f}$$

$$GR^{-1}G^T=GR^{-1}G^TQ_iQ_i^{-1}=Q_i^{-1} \tag{19}$$

$$MR^{-1}M^T=(F-F_M)^TQ_iG(G^TQ_iG)^{-1}G^TQ_i(F-F_m) \tag{20a}$$

$$=(F-F_m)^TQ_iGR^{-1}G^TQ_i(F-F_m) \tag{20b}$$

$$=(F-F_m)^TQ_i(F-F_m)=Q \tag{20c}$$

We have intentionally isolated the term $GR^{-1}G^TQ_i$, which is equal to the identity matrix in this case; its significance will become apparent later. Inserting the last three relations into Eq. (17), we get

$$PF_m+F_m^TP-PQ_i^{-1}P=0 \tag{21}$$

This can be construed as a Riccati equation resulting from optimizing a model of the form $\dot{x}_m=F_mx_m+u_m$ for a cost function consisting of no state weighting and Q_i control weighting. Assuming F_m is stable, the unique solution to this equation is $P=0$, which means $u_m=0$; thus, the model system is recovered. The feedback gain C and closed-loop system F_c of the original system become

$$C=R^{-1}(G^TP+M^T) \tag{22a}$$

$$=G^{-1}Q_i^{-1}G^{-T}\Big[G^TQ_i(F-F_m)\Big] \tag{22b}$$

$$=G^{-1}(F-F_m) \tag{22c}$$

$$F_c=F-GC=F_m \tag{23}$$

Therefore, the gain would be proportional to the available control authority and to the difference between plant and model dynamics, whereas the closed-loop dynamics would simply become those of the model. Not surprisingly, Eq. (22) is identical to Erzberger's perfect model-following control law.[2] When the integral loop is present, P is no longer zero, and we have

$$C=R^{-1}\left\{[G^T \quad 0]\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22}\end{bmatrix}+[M^T \quad 0]\right\} \tag{24a}$$

$$C=\Big[R^{-1}G^TP_{11}+G^{-1}(F-F_m) \quad R^{-1}G^TP_{12}\Big] \tag{24b}$$

$$F_c=\begin{bmatrix} F & 0 \\ H & 0\end{bmatrix}-\begin{bmatrix} G \\ 0\end{bmatrix}\Big[R^{-1}G^TP_{11}$$

$$+G^{-1}(F-F_m) \quad R^{-1}G^TP_{12}\Big] \tag{25a}$$

$$F_c=\begin{bmatrix} F_m-Q_i^{-1}P_{11} & -Q_i^{-1}P_{12} \\ H & 0\end{bmatrix} \tag{25b}$$

Hence, the gain is no longer given by Eq. (22), and the closed-loop system characteristics, as indicated by the damping and transient behavior, will be different from that of the model. Both attributes are functions of P_{12}, which is determined by the weighting matrices Q_ξ and Q_i through Eq. (16).

Consider now the term $GR^{-1}G^TQ_i$ in the more typical case with fewer controls than states,

$$GR^{-1}G^TQ_i=G(G^TQ_iG)^{-1}G^TQ_i \tag{26a}$$

$$=GG^{WL} \tag{26b}$$

where G^{WL} is the weighted left pseudo-inverse[1] of G. When the structure of the control matrix G (of rank r) is such that it contains an $r\times r$ submatrix much larger (in the two-norm sense) than the remaining submatrices, and Q_i does not change the inequality by its choice of weighting, it can be shown[16] that

$$GG^{WL}\approx I_{nr} \tag{27}$$

where I_{nr} is a matrix of dimension $n\times n$ with r ones in the diagonals and zeros everywhere else. Physically, the requirement is that r controls have a stronger effect on r state components (e.g., the angular rates) and weaker effect on the remaining states (e.g., the angles). Moreover, let $I_{nr}^\#$ be the "complement" of I_{nr}; that is, $I_{nr}^\#=(I_n-I_{nr})$, where I_n is the $n\times n$ identity matrix; then Eqs. (18) and (20) become

$$F-GR^{-1}M^T=F-GR^{-1}G^TQ_i(F-F_m) \tag{28a}$$

$$\approx F-I_{nr}(F-F_m) \tag{28b}$$

$$=I_{nr}F_m+I_{nr}^\#F=F^* \tag{28c}$$

$$MR^{-1}M^T\approx(F-F_m)^TI_{nr}Q_i(F-F_m) \tag{29}$$

If we let $Q'=Q-(F-F_m)^TI_{nr}Q_i(F-F_m)$, the Riccati equation [Eq. (13)] can be approximated as

$$PF^*+F^{*T}P-PI_{nr}Q_i^{-1}P+Q'=0 \tag{30}$$

where F^* is defined by Eq. (28). The gain and closed-loop system matrices are

$$C=G^{WL}[Q_i^{-1}P+(F-F_m)] \tag{31}$$

$$F_c=I_{nr}F_m+I_{nr}^\#F-I_{nr}Q_i^{-1}P \tag{32a}$$

$$=F^*-I_{nr}Q_i^{-1}P \tag{32b}$$

$$=I_{nr}^\#(F-F_m)+F_m-I_{nr}Q_i^{-1}P \tag{32c}$$

We note that this development bears resemblance to the perfect model-following condition as stated by Erzberger[2]

$$(GG^L-I)(F-F_m)=0 \tag{33}$$

From Eq. (28) [or Eq. (18)], we have

$$GR^{-1}M^T=GR^{-1}G^TQ_i(F-F_m)=I_{nr}(F-F_m) \tag{34}$$

or

$$0=(GR^{-1}G^TQ_i-I_{nr})(F-F_m) \tag{35a}$$

$$=(GG^{WL}-I_{nr})(F-F_m) \tag{35b}$$

The closer $GR^{-1}G^TQ_i$ is to the quasi-identity matrix I_{nr}, the better the plant behaves like the model; the closeness also affects the control restructuring strategy to be described later. As will be shown, Eq. (35a) is one of the control reconfiguration criteria. The effect of the integrator is similar to the case where G has full rank. This can be seen by computing P_{12} from Eq. (16) using $I_{nr}Q_i^{-1}$ in place of $GR^{-1}G^T$ and substituting the result into Eq. (13).

Effects of Control Reconfiguration

Reconfiguration with Variations in Control Effect

One case of interest is the degradation of a control effector. This type of failure can be approximated by reducing a column of the control matrix by a common factor. This is equivalent to an elementary row operation of matrix algebra. Let $E=\text{diag}[1 \quad 1\cdots\alpha\cdots 1]$, $\alpha>0$, where α is the factor by which the effectiveness is reduced. If G' $(=GE)$ is the new control matrix, the weighting matrices and Eqs. (18-20) then become

$$R' = G'^T Q_i G' = E^T G^T Q_i G E = E^T R E \tag{36}$$

$$M' = (F - F_m) Q_i G' = ME \tag{37}$$

$$(F - G'R'^{-1}M'^T) = (F - GEE^{-1}R^{-1}E^{-T}E^T M) \tag{38a}$$

$$= (F - GR^{-1}M^T) \tag{38b}$$

$$G'R'^{-1}G' = GEE^{-1}R^{-1}E^{-T}E^T G = GR^{-1}G^T \tag{39}$$

$$M'R'^{-1}M'^T = MEE^{-1}R^{-1}E^{-T}E^T M = MR^{-1}M^T \tag{40}$$

Hence, there is no change in the Riccati equation (Eq. 13 or Eq. 17). As expected, the gain is altered since R is decreased so as to allow additional control activity, but the closed-loop system remains unchanged:

$$C' = R'^{-1}(G'^T P + M'^T) \tag{41a}$$

$$= E^{-1}R^{-1}E^{-T}(E^T G^T P + E^T M) = E^{-1}C \tag{41b}$$

$$F_c' = F - G'C' = F - GEE^{-1}C = F - GC = F_c \tag{42}$$

Since the integral effect enters only through P_{12} [Eq. (16)], which is unaffected by this type of failure, the analysis holds. The control restructuring process is now clear. When the gains are recalculated with G', the original system performance is restored. Note that these results are independent of the invertibility of G.

If $\alpha = 0$, reconfiguration is addressed simply by reducing the control vector dimension and adjusting G, R, and M accordingly. In this situation, it is important to insure that controllability has been retained with the reduced-dimension controller.

The effects of single- and multiple-element changes are more difficult to analyze since they no longer can be represented as elementary matrix operations. However, Eq. (26a) [or equivalently Eq. (35a)] can still be computed using G' $(=G+\delta G)$ in place of G. As long as the equality [Eq. (27)] (approximately) holds, the controls can force the plant to behave like the model. Otherwise, the control matrix must have undergone substantial changes, such that the original performance cannot be approximated by the implicit model-following strategy.

Reconfiguration with Variations in System Dynamics

System dynamics could change, for example, as a consequence of structural damage. We first examine the reconfiguration when a row in F changes by a common factor. Row changes are equivalent to premultiplication by the elementary matrix E; that is, $F' = EF$. If G is invertible, then the previous analysis holds, and the performance can be recovered by simply recalculating the gain using F' instead of F. For nonsquare G, when the changes are along the nonzero rows of I_{nr}, Q and M are modified such that the solution of the Riccati equation [Eq. (13)] is not significantly affected. In this case, the gain and the closed-loop system are

$$C = R^{-1}(G^T P + M'^T) \tag{43a}$$

$$\approx G^{WL}[Q_i^{-1}P + (F' - F_m)] \tag{43b}$$

$$F_c' \approx F' - GC \approx EF - GG^{WL}Q_i^{-1}P - GG^{WL}(EF - F_m) \tag{44a}$$

$$\approx I_{nr}F_m - I_{nr}^{\#}EF - I_{nr}Q_i^{-1}P \tag{44b}$$

$$= I_{nr}^{\#}(F' - F_m) + F_m - I_{nr}Q_i^{-1}P = F_c \tag{44c}$$

Thus, the gain is altered, but the closed-loop system remains essentially the same. As can be seen from Eqs. (32) and (44), the reconfiguration may not recover the performance if F is changed along the zero rows of I_{nr}; that is, the following relation must hold:

$$I_{nr}^{\#}(F' - F_m) \approx I_{nr}^{\#}(F - F_m) \tag{45}$$

This equation constitutes the other reconfiguration criterion to be applied in the case of system dynamic changes. Note that since P_{12} does not depend on F, changes in the system dynamics have no effect on the integral loop.

Failures are unlikely to cause alterations in the manner described. Single- or multiple-element failures by different factors are expected. If the reconfiguration criteria are satisfied, then the performance will be recovered when the gains are recalculated using F' $(=F+\delta F)$.

We point out here that reconfiguration is possible with $GR^{-1}G^T Q_i$ not close to I_{nr}, although acceptable performance may require it. If the expression exhibits a nonidentity matrix structure, then the model is not well followed; consequently, the performance of the reconfigured system would not be near its nominal values. Moreover, since P_{12} does not depend on F, changes in the system dynamics have no effect on the integral loop; therefore the analysis carries over to the PIIMF control law. This and other observations are verified numerically in the next section.

Simulation Results

The analysis described in the previous sections is applied to the lateral-axis model of the F-4 aircraft. The state vector consists of roll rate p (rad/s), yaw rate r (rad/s), sideslip β (rad), and roll angle ϕ (rad); i.e., $\boldsymbol{x} = [p\ r\ \beta\ \phi]^T$. The controls are rudder δr(in.) and aileron δa(in.); i.e., $\boldsymbol{u} = [\delta r\ \ \delta a]^T$. The system, control, and model matrices are[6]

$$F = \begin{bmatrix} -1.768 & 0.415 & -14.25 & 0.0 \\ -0.0007 & -0.3831 & 6.038 & 0.0 \\ 0.0016 & -0.9975 & -0.1551 & 0.0586 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{46}$$

$$G = \begin{bmatrix} 1.744 & 8.952 \\ -2.92 & -0.3075 \\ 0.0243 & -0.0036 \\ 0.0 & 0.0 \end{bmatrix} \tag{47}$$

$$F_m = \begin{bmatrix} -4.0 & 0.865 & -10.0 & 0.0 \\ 0.04 & -0.507 & 5.87 & 0.0 \\ 0.0016 & -1.0 & -0.743 & 0.0586 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{48}$$

$$Q_i = \mathrm{diag}[0.01 \quad 0.01 \quad 0.01 \quad 0.01] \tag{49}$$

$$Q_\xi = \mathrm{diag}[1.0 \quad 1.0] \tag{50}$$

The speed of response is determined by the ratio of Q_i to Q_ξ. The weighting matrices as shown are chosen to produce reasonable tracking; no extensive search for other combinations was conducted.

For this system, Eq. (26a) is found to be

$$GR^{-1}G^TQ_i = \begin{bmatrix} 1.0 & -6.34\times 10^{-6} & -0.0007 & 0.0 \\ -6.256\times 10^{-6} & 1.0 & -0.00874 & 0.0 \\ -0.0007 & -0.00874 & 7.7\times 10^{-5} & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{51}$$

This can be seen to be approximately equal to I_{22}. Therefore, two model states (p and r) can be followed, and the reconfiguration is expected to work well for changes that affect primarily these two variables.

The time histories of the rudder, aileron, sideslip, and roll angle subject to a 1 rad roll-angle command and 0.5 rad sideslip (a rather unusual maneuver) are used as performance evaluations. Several failure cases are examined here.

Case 1

Aileron effectiveness is reduced by half; that is, the second column of G is multiplied by 0.5. This failure is equivalent to a loss of aileron surface area that decreases the available rolling control moment. The failed response is shown as a broken curve in Fig. 1. The reconfigured system response is shown as the bold-face curve. Notice that the yaw and roll angle responses of the failed and unfailed systems are indistinguishable, whereas the aileron exhibits higher activity with no visible change in rudder motion. Corresponding gains C_1 and C_2 for this and other cases are listed in Table 1.

Case 2

The first row of the F matrix and the second column of G are reduced by half. This corresponds to an overall decrease in

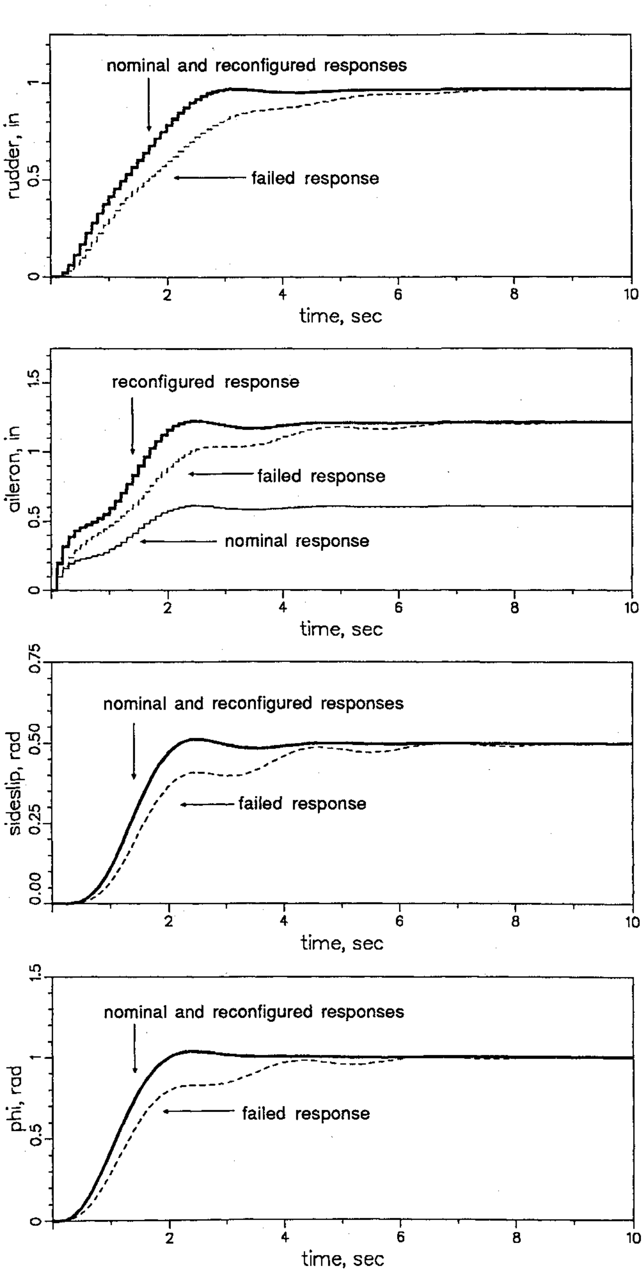

Fig. 1 Control and state responses of the failure case 1.

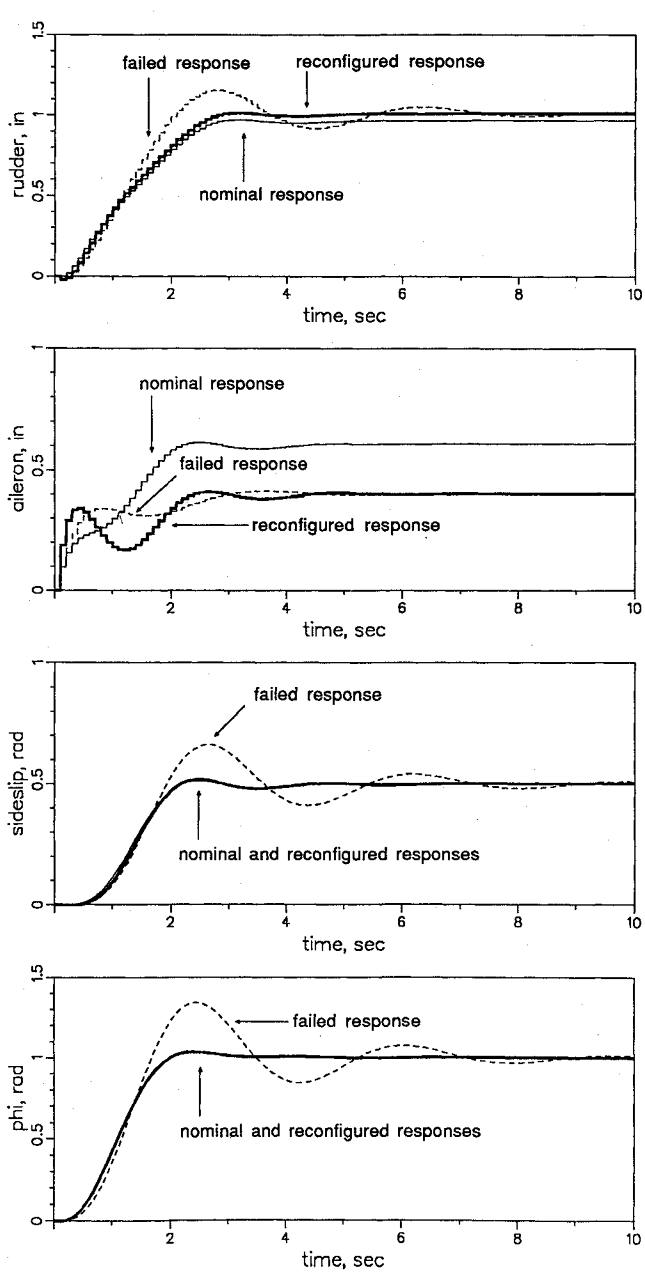

Fig. 2 Control and state responses of the failure case 2.

Table 1 Control gains for sample scenarios

Scenario	C_1				C_2	
No failure	−0.3892	−1.07871	1.99	−1.9472	2.9054	−1.95
case	0.517	0.272	−1.1484	1.36625	0.024	1.3282
Failure	−0.3892	−1.0787	1.99	−1.9472	2.9054	−1.95
case 1	1.034	0.544	−2.3	2.7325	0.048	2.656
Failure	−0.4	−1.077	1.909	−1.948	2.905	−1.95
case 2	1.235	0.497	−0.6733	2.7325	0.0485	2.656
Failure	−22.181	−32.4	108.94	−122.3106	147.287	−152.09
case 3	−3.741	6.331	−21.806	23.696	−27.914	30.43
Failure	−0.312	−0.795	4.16	−1.477	3.28	−1.217
case 4	0.53	0.1893	−1.823	1.417	−0.286	1.297

aerodynamic forces and control effectiveness that contribute to the roll rate, as might result from battle damage to the wings. The failed and reconfigured responses are shown in Fig. 2. Note that the control activity is less in the reconfigured case since the ratio of control to the aerodynamic effectiveness is larger than that of the nonfailed system.

Case 3

The (2,1) element of G (proportional to the aileron yaw effectiveness $C_{n_{\delta a}}$) is reduced from -2.92 to -0.1 (a significant change). This cross-coupling alteration may be one of the many other changes induced by damages; it is isolated here to illustrate its effect. The responses are shown in Fig. 3. For this failure, $GR^{-1}G^TQ_i$ is:

$$GR^{-1}G^TQ_i = \begin{bmatrix} 1.0 & -0.0003 & -0.0012 & 0.0 \\ -0.0003 & 0.934 & -0.2484 & 0.0 \\ -0.0012 & -0.2484 & 0.0661 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{52}$$

Because the expression departs from its quasi-identity-matrix structure, the reconfiguration is unable to recover the original response.

Case 4

The third row of the system matrix is reduced by a factor of 0.25. The responses are shown in Fig. 4. For this case, Eq. (45) no longer holds, and we expect the reconfiguration to perform poorly. The integral compensator still forces zero steady-state error, albeit at a much longer time constant. Note that this is not a physically realizable case since it involves changing a pure integration factor. It is used here to illustrate the importance of reconfiguration criterion 2. A more realistic scenario and additional cases can be found in Ref. 16.

A few additional observations are due. $GR^{-1}G^TQ_i$ remained close to I_{22} despite the changes in the first two failure cases where control restructuring performed well. When aileron effectiveness was reduced, the reconfigured system simply increased the aileron motions with no apparent change in rudder activity. This is because the rudder is an ineffective surface for roll control at this flight condition. In a more cross-coupled system, the control activities would be more evenly reallocated. To reduce control motions, control-limiting criteria can be incorporated into the cost function.

Similarly, model uncertainty can be taken into account by loop-shaping techniques at the outset; however, robustness properties are altered by control reconfiguration. The integral effect is largely unaffected by the failure, even though the ξ response is different due to different plant dynamics. No updating of Q_ξ is necessary if the ratio of the integration rate and error feedback is to be preserved. Complete control failures were not illustrated, but it can be shown (see Ref. 16) that system performance cannot be totally restored, and the system behavior will depend on how well the remainder of the controls can achieve the model following.

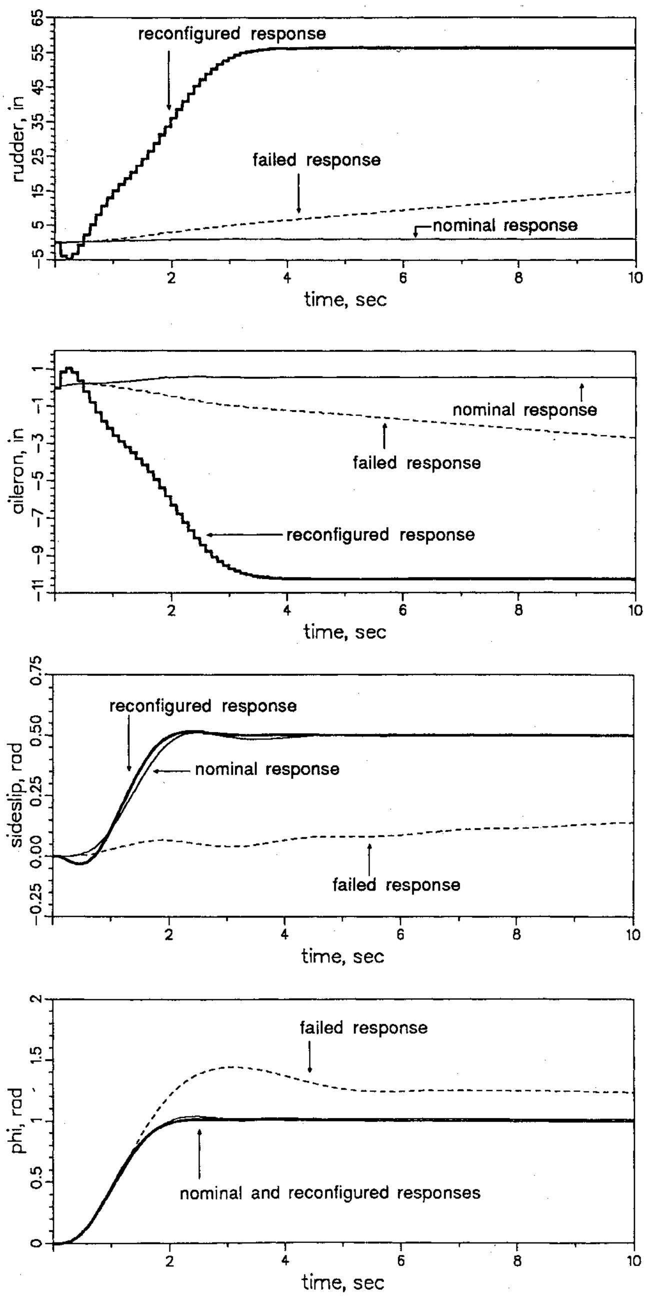

Fig. 3 Control and state responses of the failure case 3.

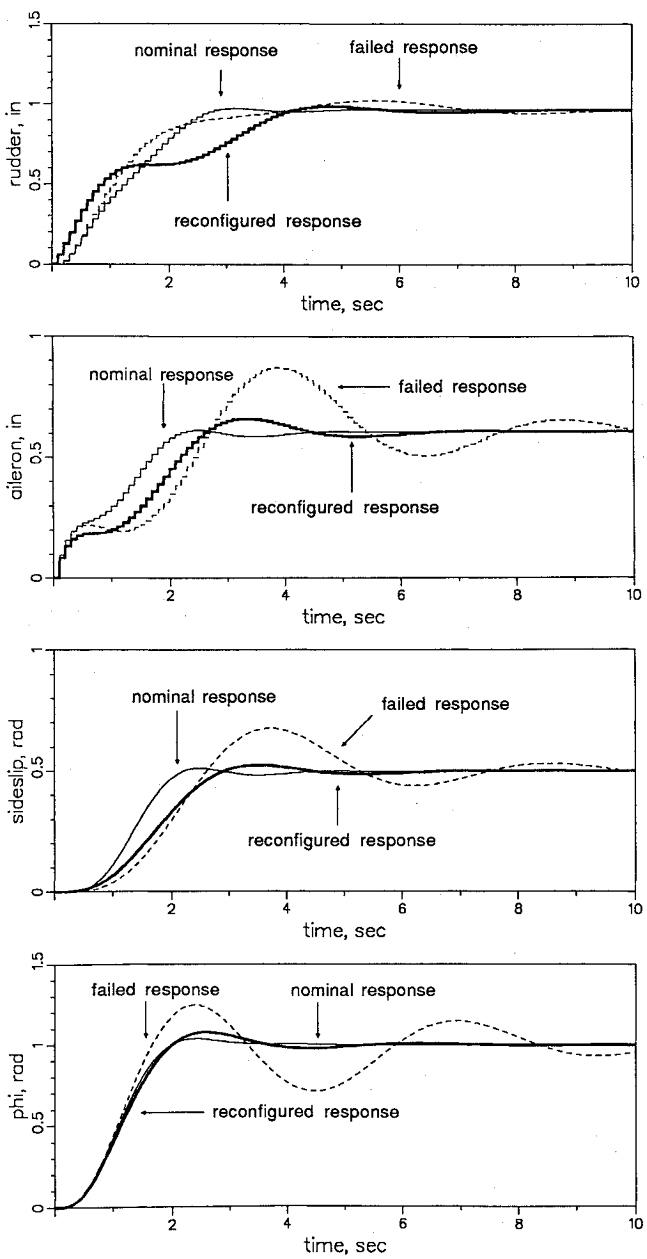

Fig. 4 Control and state responses of the failure case 4.

Conclusions

An implicit model-following control strategy for restructuring a control system following control-effector failures and alterations in system dynamics has been presented. Examination of the inherent properties of the control scheme led to the formulation of two reconfiguration criteria that, if satisfied, guarantee the performance recovery of a failed system when the feedback gains are recalculated using the new dynamics. Care must be exercised in applying the strategy since, in some cases, higher demand is placed on the remaining control authority. However, when properly used, the proportional-integral implicit model-following control law is a good candidate for control reconfiguration.

The implicit model-following strategy reconfigures the system by rearranging the weighting matrices to preserve closed-loop characteristics. This is analogous to a pole-placement technique that assigns the closed-loop poles to the same locations before and after the failure. However, implicit model following does more than positioning eigenvalues and matching desired eigenvectors; it also addresses command response, which is not totally specified by eigenvalues and eigenvectors. The advantages of the model-following scheme are that, following partial control failure or changes in the system dynamic characteristics, it attempts to maintain the desired eigenstructure and command response, and at the same time, it allows tradeoffs between control activity and system response.

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